 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

 **M.Sc.** DEGREE EXAMINATION - **STATISTICS**

THIRD SEMESTER – NOVEMBER 2012

# ST 3812 - STOCHASTIC PROCESSES

 Date : 03/11/2012 Dept. No. Max. : 100 Marks

 Time : 9:00 - 12:00

**SECTION – A**

**Answer all the questions: (10 x 2 = 20 Marks)**

1. Define a point process.
2. Define n step transition probability.
3. Write any two basic properties of the period of a state.
4. If i ↔ j and if i is recurrent then show that j is also recurrent.
5. Define mean recurrence time.
6. What is the infinitesimal generator of a birth and death process?
7. Define excess life and current life of a renewal process.
8. Define a sub martingale.
9. Write down the postulates of a birth and death process.
10. Write down any two examples for stationary process.

**SECTION – B**

**Answer any Five questions: (5 x 8 = 40 Marks)**

1. Explain (i) process with stationary independent increments (ii) Markov processes.
2. Explain spatially homogenous Markov chains.
3. Prove that a state i is recurrent if an only if

 ∑ Piin = ∞

1. For a two dimensional random walk, prove that ∑ P00n = ∞
2. Determine stationary probability distribution for a random walk whose transition probability matrix is

 0 1 0 0 . . .

 q1 0 p1 0 . . .

 0 q2 0 p2 . . .

 P = .

 .

 .

-2-

1. Derive Pn (t) for a Poisson process.
2. Derive the expected value of a birth and death process with linear growth and immigration.
3. State and prove the basic renewal theorem.

**SECTION – C**

**Answer any two questions: (2 x 20 = 40 Marks)**

1. (a) State and prove the basic limit theorem of Markov chains.

(b) Explain discrete renewal equation. (15 + 5)

1. (a) Derive the differential equations for a pure birth process.

(b) Derive the Kolmogorov forward and backward differential equations of a birth and

 death process. (10 +10)

1. (a) Explain renewal function, excess life, current life and mean total life.

(b) If {Xt}is a renewal process with μ = E [Xt] < ∞ , then show that

 lim 1/t M (t) = 1/μ as t → ∞ (8 + 12)

1. (a) Show that π is the smallest positive root of the equation ϕ(s) = s

 for a branching process.

(b) Compute expectation and variance of branching process. (10 + 10)

\*\*\*\*\*\*